



Lecture 12 Single-qubit gates and discrete quantum tomography

- Single-qubit gate
- discrete quantum tomography

Quantum Information

- Superposition
- entanglement (Superposition of product states)
- processor
- memory (register)

1 qubit = $\begin{pmatrix} |0\rangle \\ |1\rangle \end{pmatrix}$ $\xrightarrow{\text{Superposition}}$ $\cos(\theta) |0\rangle + e^{i\phi} \sin(\theta) |1\rangle$

1 classical bit $\begin{cases} |0\rangle \rightarrow R \\ |1\rangle \rightarrow L \end{cases}$

$\theta = \frac{\pi}{2}, \phi = \frac{\pi}{2} \rightarrow \frac{1}{\sqrt{2}} |0\rangle + \frac{i}{\sqrt{2}} |1\rangle \rightarrow R$

$\theta = \frac{\pi}{2}, \phi = \frac{3\pi}{4} \rightarrow \frac{1}{\sqrt{2}} |0\rangle + \frac{1}{\sqrt{2}} \left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right) |1\rangle = \frac{1}{2} |0\rangle + \frac{1+i}{2} |1\rangle \rightarrow L$

2 qubit $\begin{cases} |00\rangle \rightarrow 0 \\ |01\rangle \rightarrow 1 \\ |10\rangle \rightarrow 2 \\ |11\rangle \rightarrow 3 \end{cases} \xrightarrow{\text{unitary gate = transformation}} \frac{1}{\sqrt{3}} (|00\rangle + |1\rangle + |11\rangle) =$

total probability remains 1

$\hat{U} |\psi\rangle = |\psi'\rangle \rightarrow \hat{U}^{-1} |\psi'\rangle = |\psi\rangle$

$\hat{U} \hat{U}^\dagger = \hat{I} = \hat{U}^\dagger \hat{U}$

$\hat{U}^{-1} = \hat{U}^\dagger$

Diagram illustrating a function $f(x)$ and its inverse $f^{-1}(y)$. The function f maps x to y . The inverse function f^{-1} maps y back to x . The diagram shows x and y as inputs/outputs and f and f^{-1} as boxes representing the functions.





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NMR
 $\frac{d\vec{M}}{dt} = \vec{M} \times \vec{B}$
 $\rightarrow \frac{d\vec{S}}{dt} = \vec{\Omega} \times \vec{S} = \vec{S} \times (-\vec{\Omega})$

Rotation Transformation

$\theta = 0 \rightarrow \vec{\Omega} = (\Omega, 0, 0) \xrightarrow{\text{rotation}} \vec{Q} = (\Omega \cos \phi, \Omega \sin \phi, 0)$

$|0\rangle \rightarrow \frac{1}{\sqrt{2}}(|0\rangle + e^{i\phi} \sin(\frac{\theta}{2})|1\rangle)$

$\rightarrow C_0|0\rangle + C_1|1\rangle$

universal gate set

$\{X, \text{PI}/8, H, \text{CNOT}\}$

$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$
 $|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

X gate $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ \rightarrow

$|0\rangle \rightarrow |1\rangle$ (NOT)

PI/8 gate $\begin{pmatrix} 0 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$ \rightarrow

$e^{i\pi/4}$

Hadamard gate $\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$ \rightarrow

$|0\rangle \rightarrow \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$
 $|1\rangle \rightarrow \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

Hadamard French

$\phi_0 = \frac{\pi}{2} + \phi$
 $\tau = \frac{\pi}{2} \frac{1}{\Omega}$

$\phi_0 = 0 \rightarrow \vec{Q} = (\Omega, 0, 0)$

$\theta = \pi, \tau = \frac{T}{2} = \frac{\pi}{2\Omega}$
 $\theta = \frac{\pi}{2}, \tau = \frac{T}{4} = \frac{\pi}{4\Omega}$





✓ - Single-qubit gate

- discrete quantum tomography
obtaining the full information.

$\rho = \begin{pmatrix} \frac{1+W}{2} & \frac{u-iv}{2} \\ \frac{u+iv}{2} & \frac{1-W}{2} \end{pmatrix}$

$\hat{P}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$\hat{P}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$\hat{P}_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

ODMR
 $P_0 = P_1$
 $\begin{pmatrix} 1-W & 0 \\ 0 & 1+W \end{pmatrix}$
 $-W$

$\vec{s}_2 = (u, 0, 0) \rightarrow \hat{P}_y(\frac{\pi}{2})$

$\vec{s}_2 = (0, v, 0) \rightarrow \hat{P}_x(\frac{\pi}{2})$

$\vec{s}_3 = (0, 0, w) \rightarrow \hat{P}_z$

$\mathcal{J} = (u, v, w)$

⑧

$|0\rangle \rightarrow \hat{P}_z \rightarrow w$

$|0\rangle \rightarrow \hat{P}_y(\frac{\pi}{2}) \rightarrow \hat{P}_x(\frac{\pi}{2}) \rightarrow u$

$\cos(\omega t) \rightarrow \hat{P}_x \rightarrow \hat{P}_y(\frac{\pi}{2}) \rightarrow \hat{P}_x(\frac{\pi}{2}) \rightarrow v$

\hat{P}_x



